

Local equivalence of topological order and decoding algorithms

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Joint work with: G. Duclos-Cianci and H. Bombin

Sydney quantum information theory workshop
Coogee, Australia, January 2011

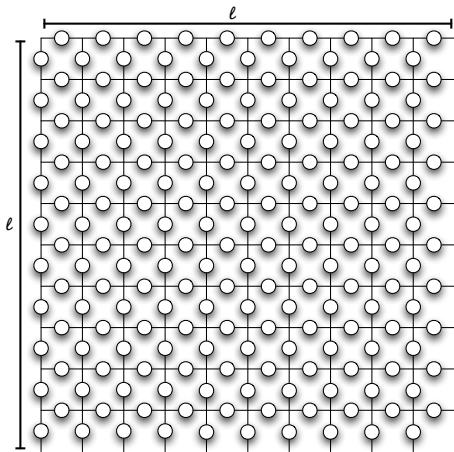
Outline

- 1 Kitaev's code
 - Definition
 - Hamiltonian's symmetries
- 2 Errors: defect creation, diffusion, and annihilation
- 3 Decoding problem
 - Task description
 - Perfect matching algorithm
- 4 Renormalization Group Decoder
 - Coarse graining
 - Mean-Field Equations
- 5 Results for Kitaev's code
- 6 Local equivalence

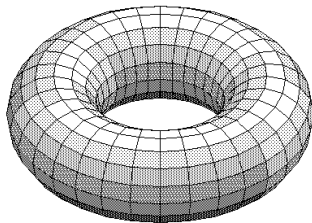
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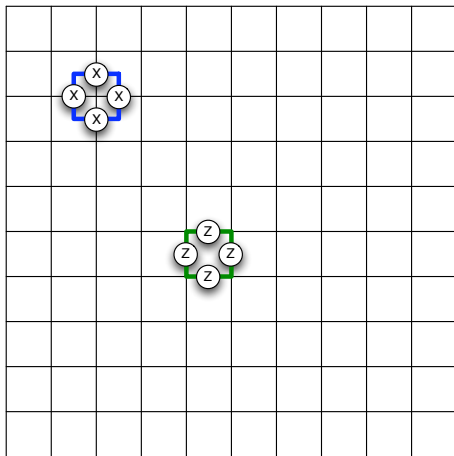
Lattice



- Two-dimensional square lattice
- Periodic boundary conditions



Hamiltonien



- Site operator:

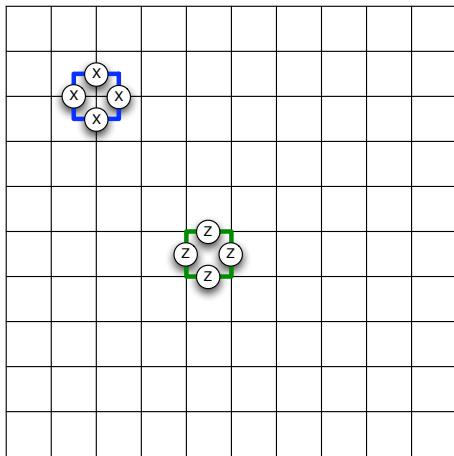
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- Plaquette operator:

$$B_p = \prod_{i \in v(p)} \sigma_z^i$$

- $H = -(\sum_s A_s + \sum_p B_p)$

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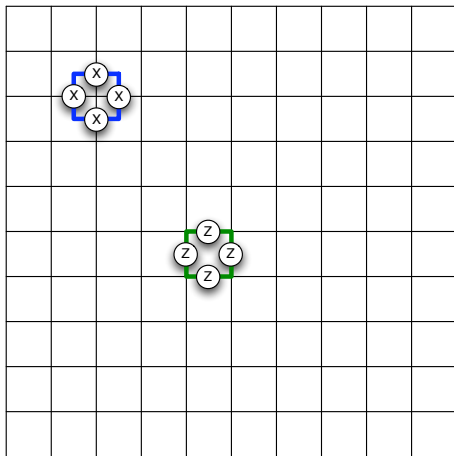


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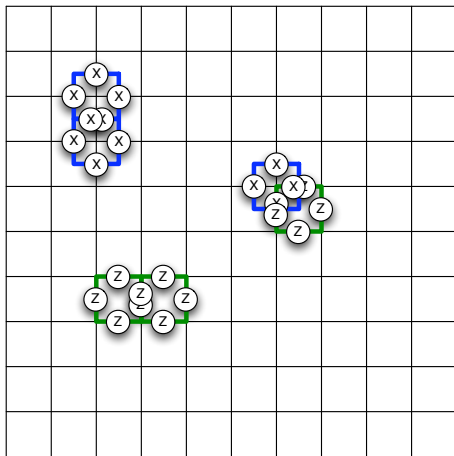


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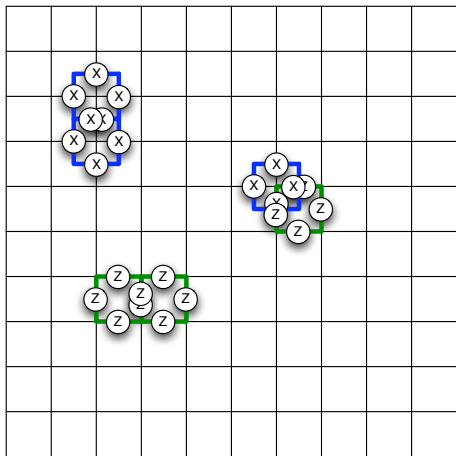
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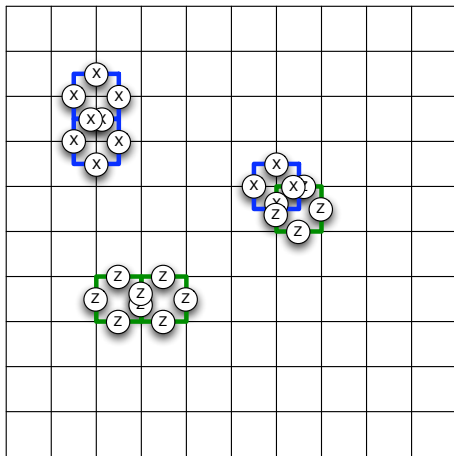
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- $[A_s, A_{s'}] = [B_p, B_{p'}] = 0$
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- The Hamiltonian is a sum of commuting terms.
 - Exactly solvable
 - Constant gap

Hamiltonian



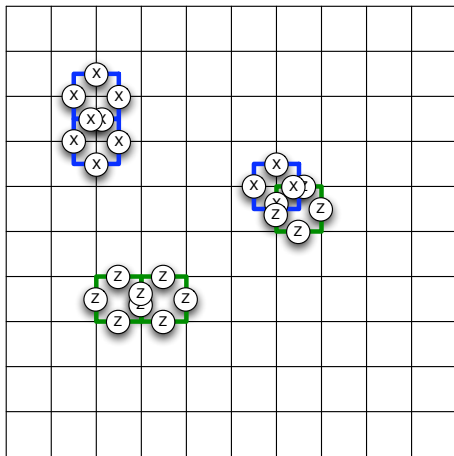
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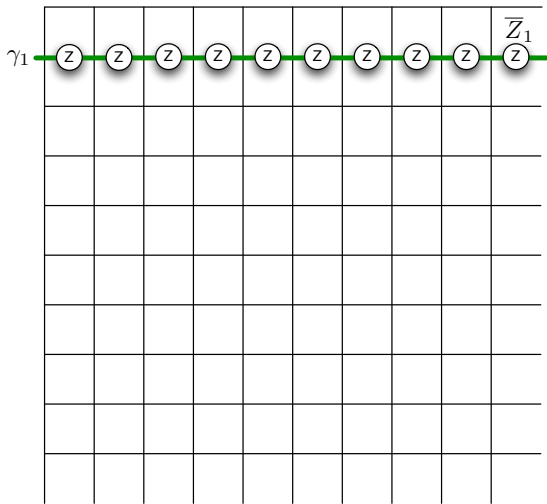
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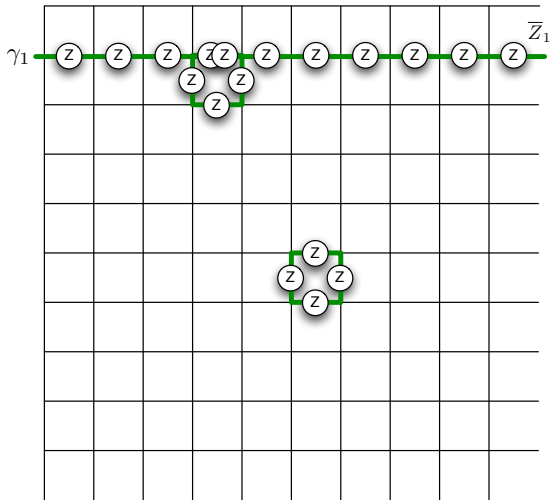
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- $\bar{Z}_1 = \prod_{i \in \gamma_1} \sigma_z^i$
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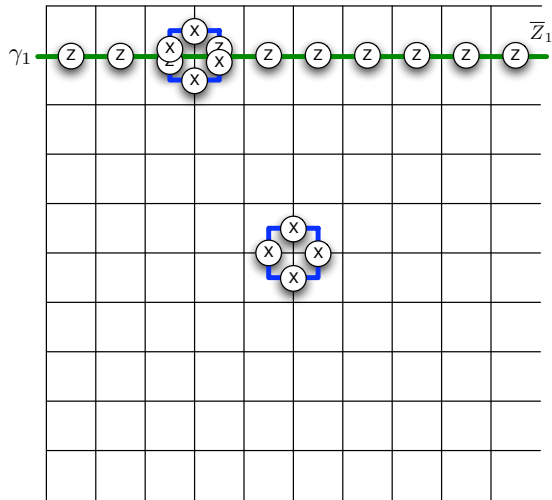


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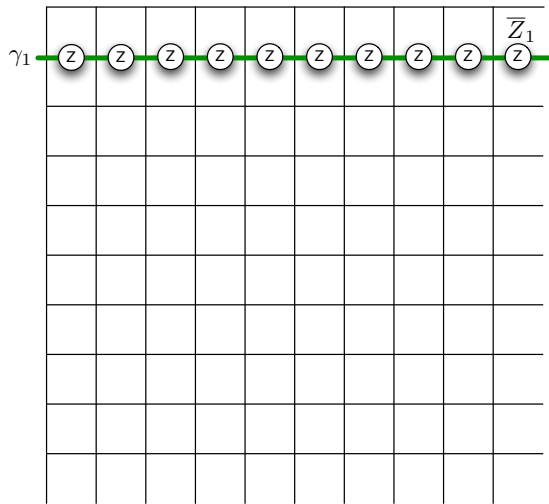


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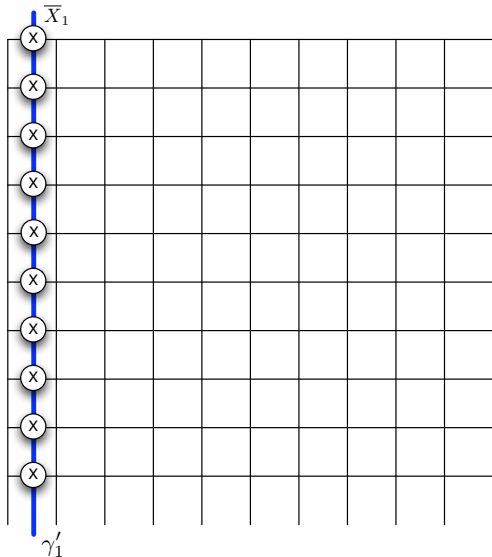
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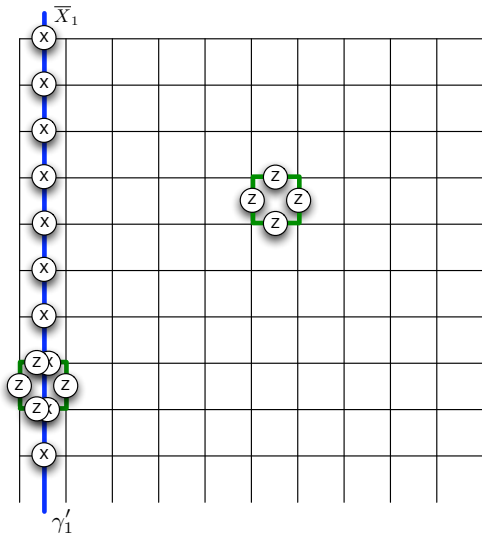
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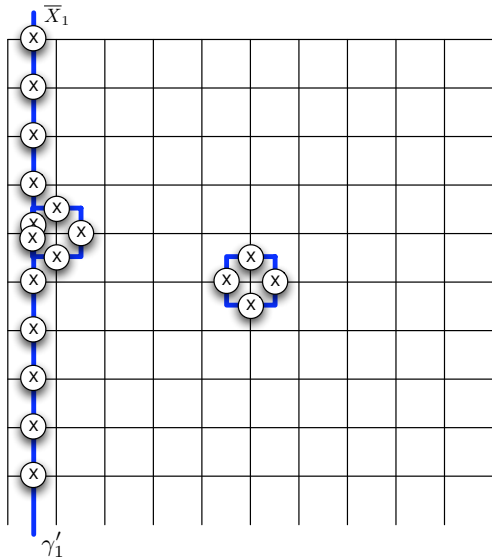
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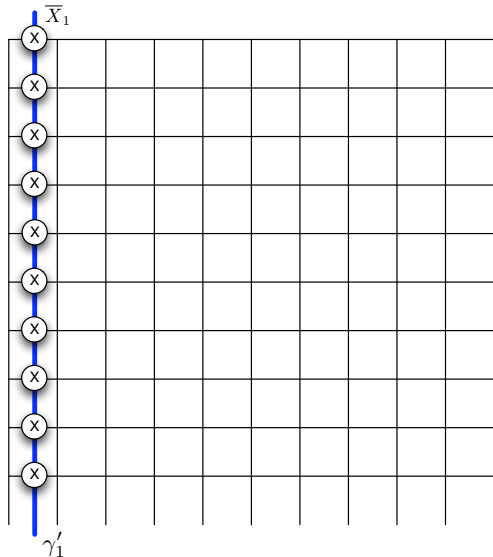
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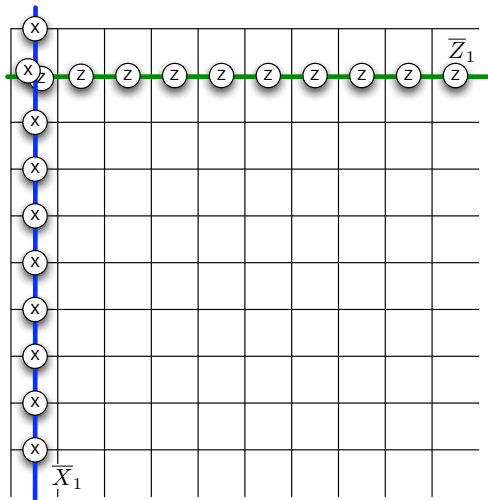
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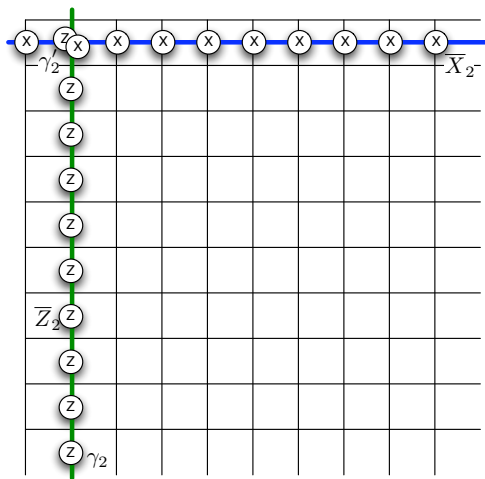


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- By reflecting around the diagonal, we obtain two new symmetry operators

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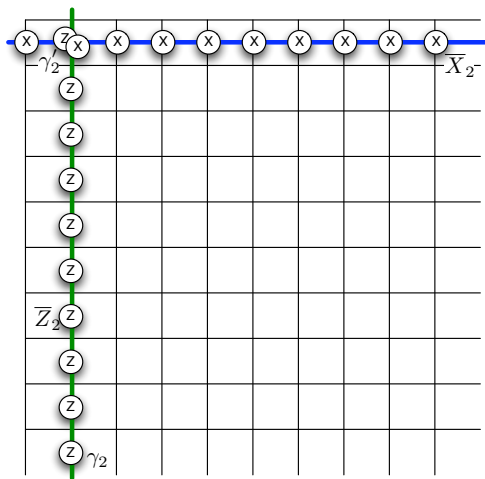
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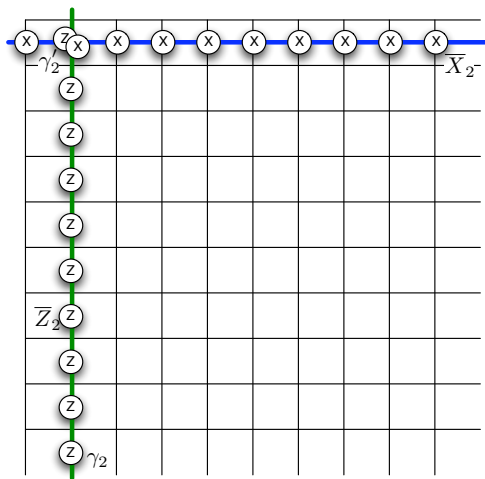
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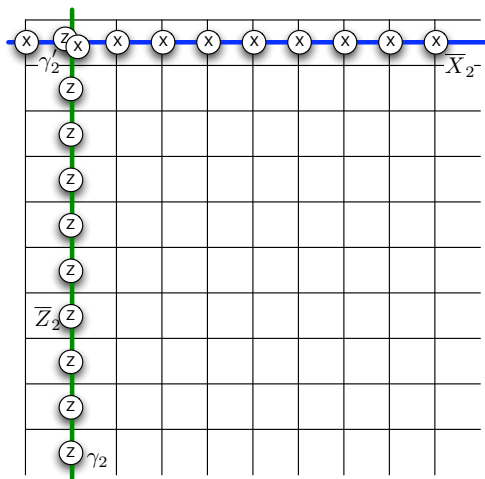
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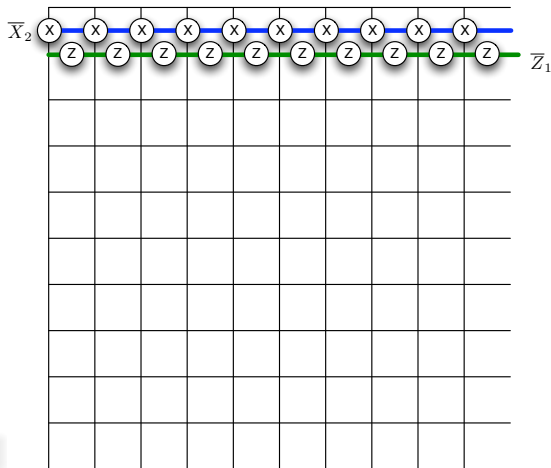
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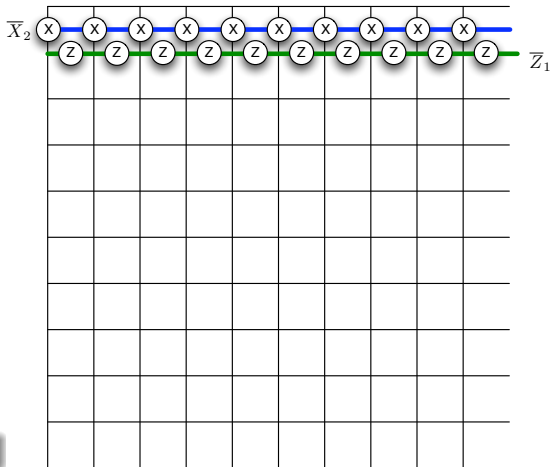


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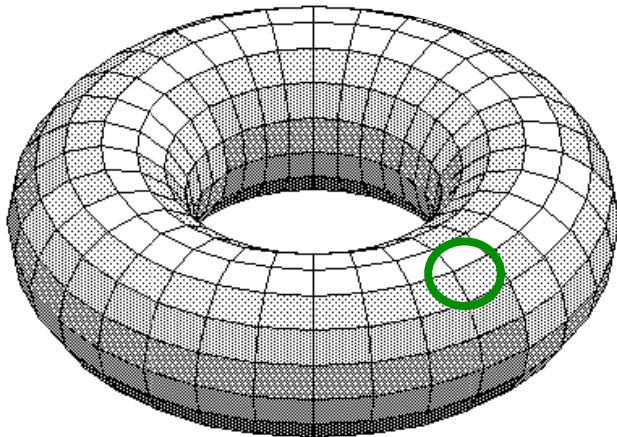
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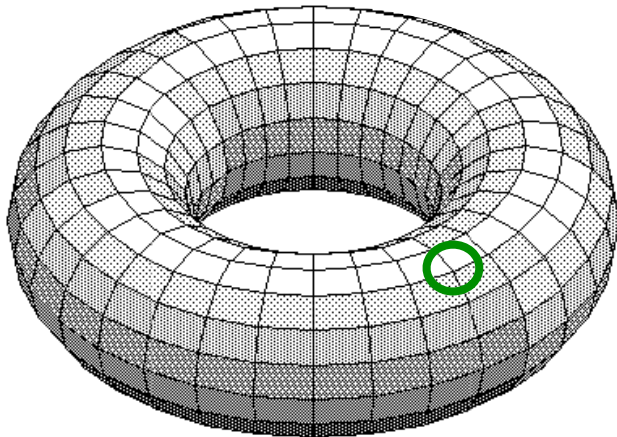
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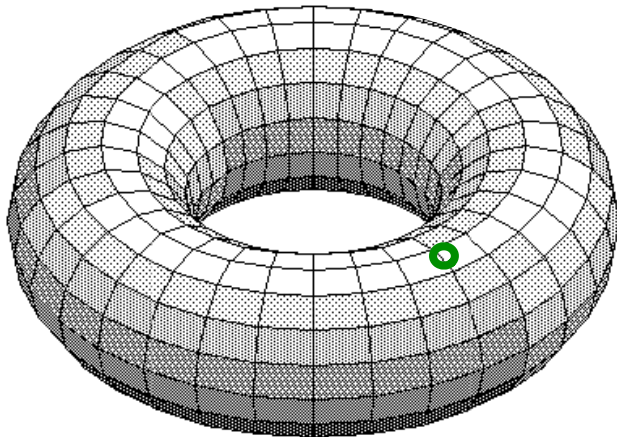
Trivial cycles



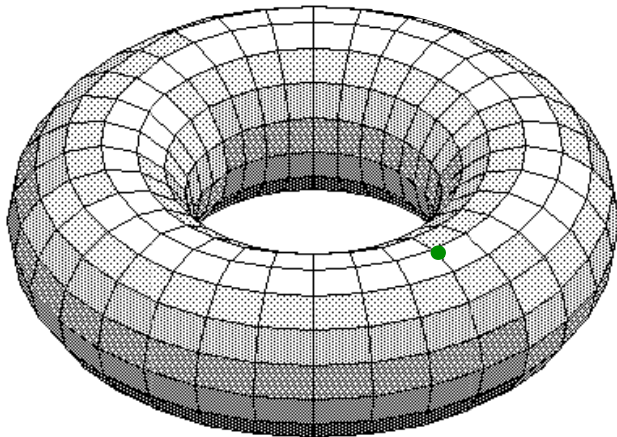
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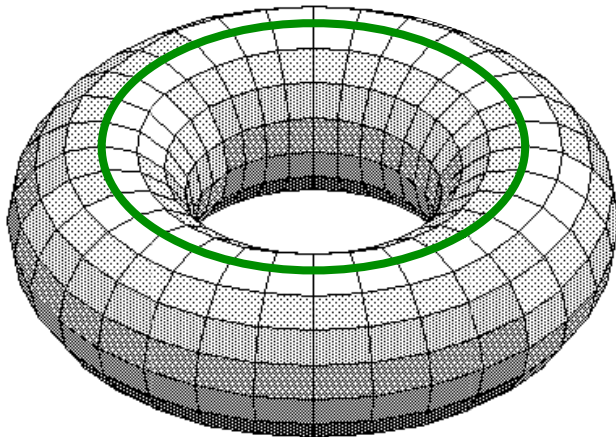
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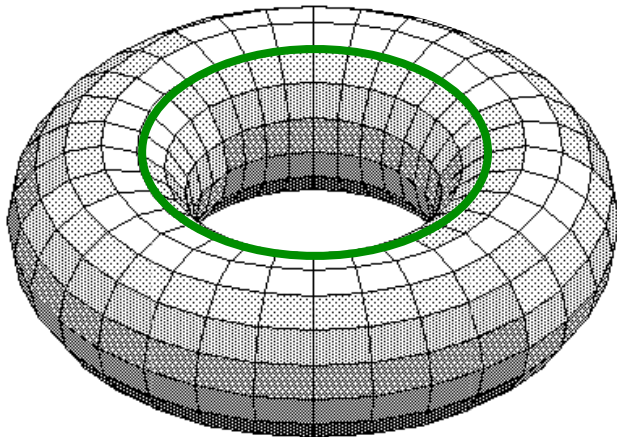
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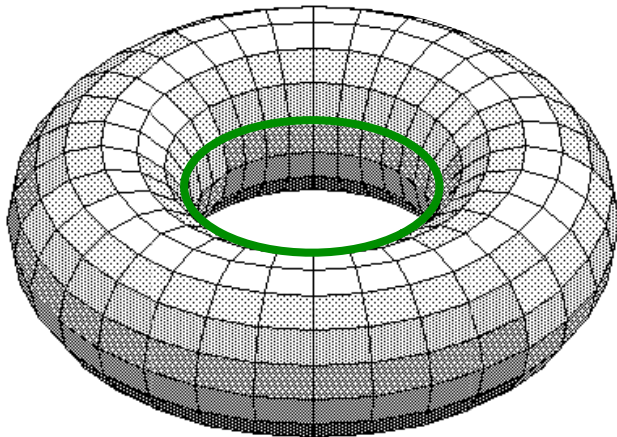
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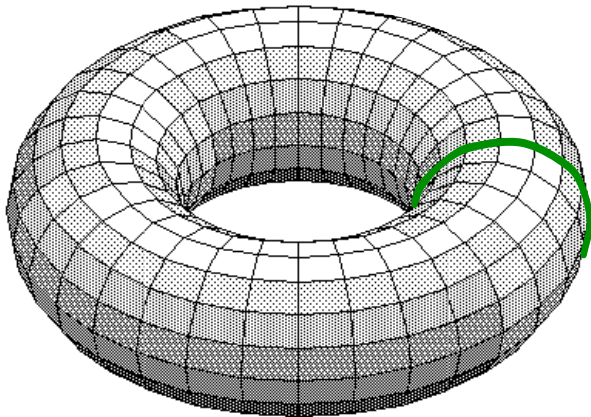
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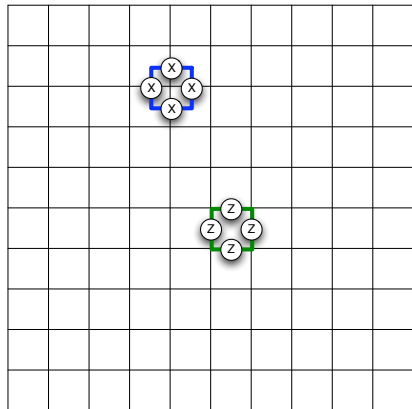
Non-trivial cycles



Trivial cycles and ground space

- $H = -(\sum_s A_s + \sum_p B_p)$
- The A_s et B_p are trivial cycles
- Trivial action on ground space
 $A_s|\psi\rangle = B_p|\psi\rangle = +1|\psi\rangle$
- A_s B_p generate all trivial loops.

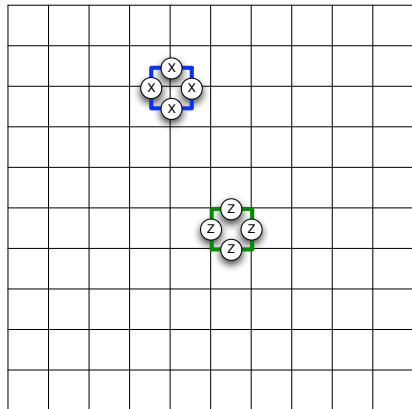
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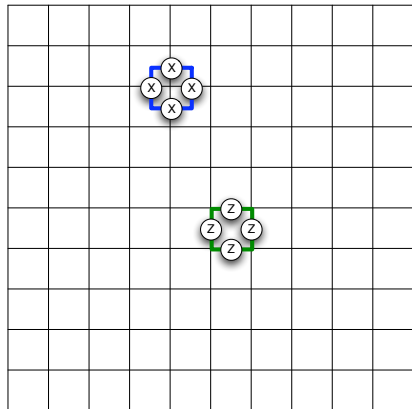
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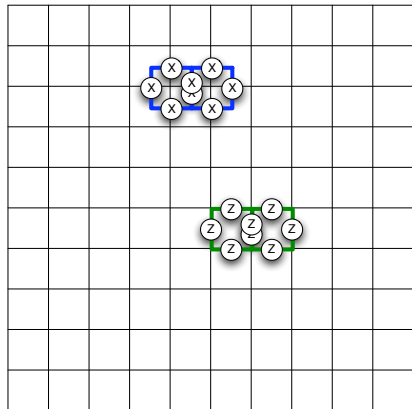
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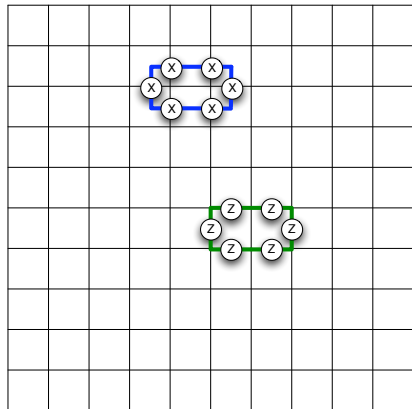
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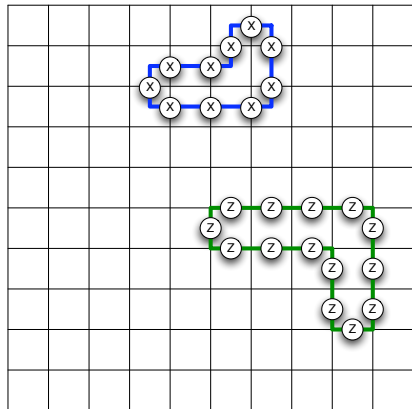
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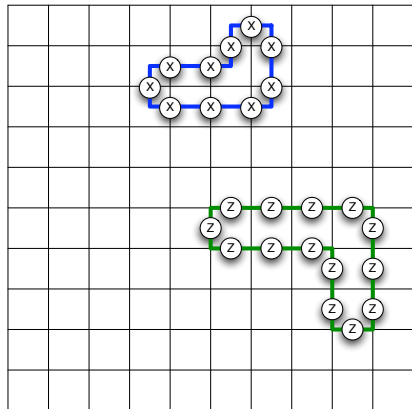
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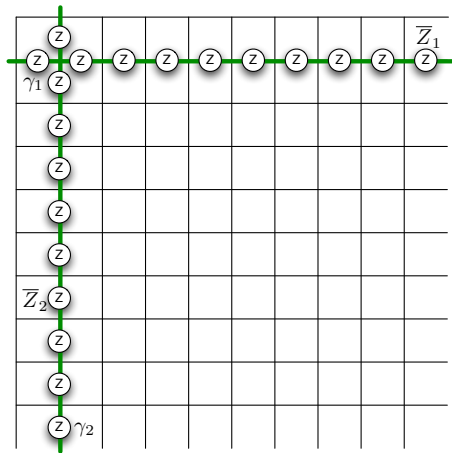
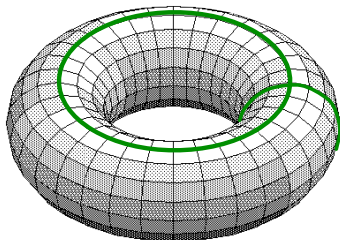
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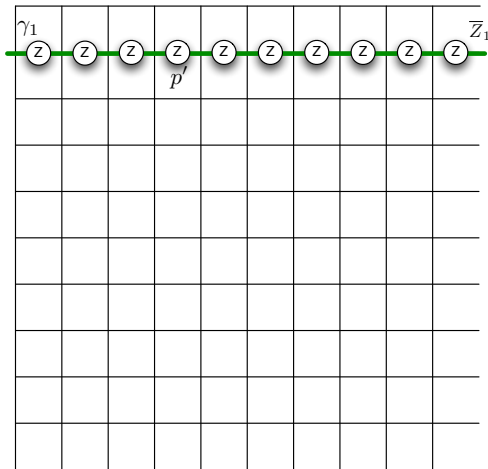
Non-trivial cycles

- γ_1 and γ_2 wrap around the torus: they are non-trivial cycles



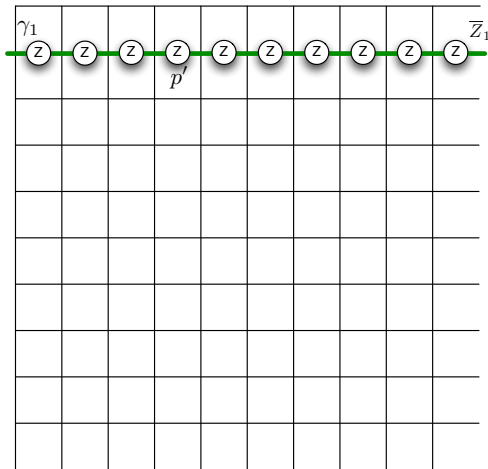
Gauge choice

- $|\psi\rangle = B_{p'}|\psi\rangle$
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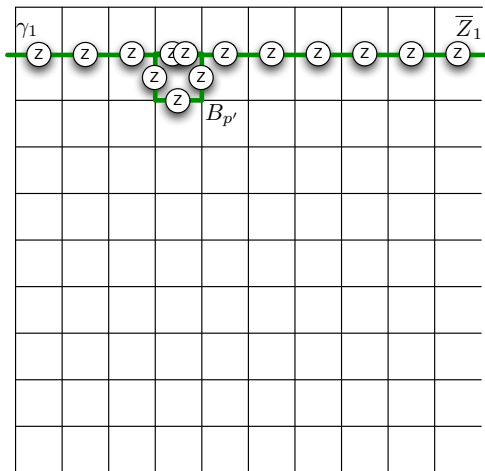
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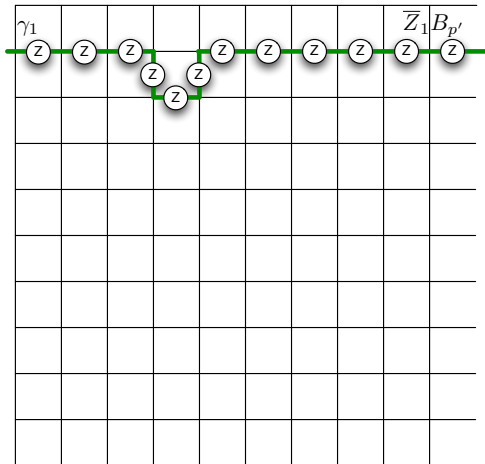
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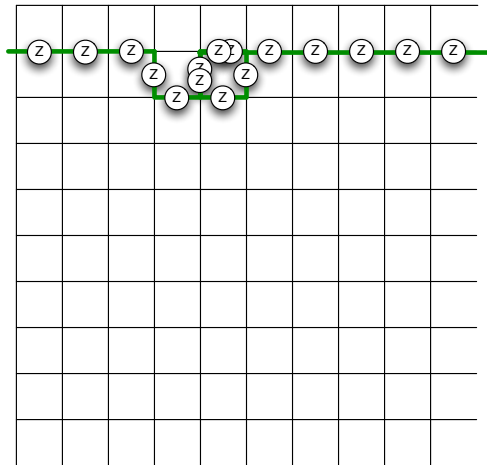
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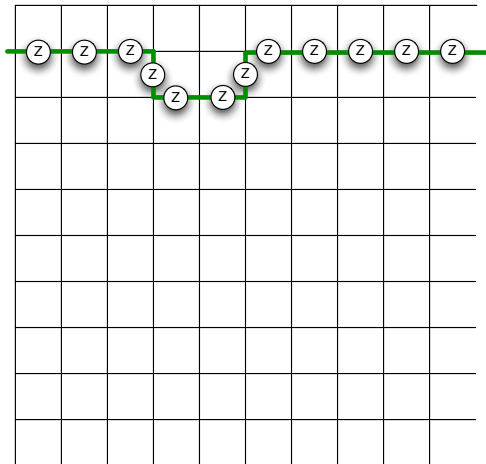
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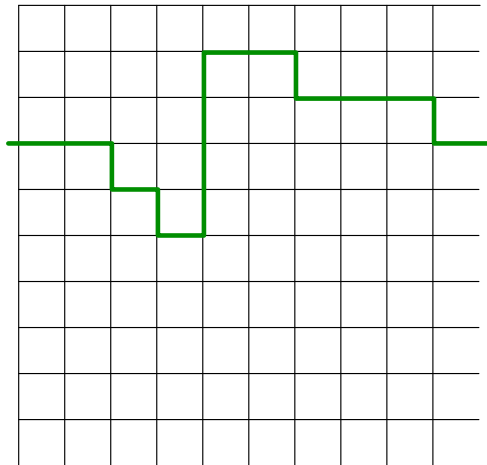
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Hamiltonian - Topology

- One degree of freedom associated to each non-trivial cycle.
- Operator in same homological class act identically on ground space.
- We encode the quantum information is those degrees of freedom:
 - The information can only be modified by topologically non-trivial operators.
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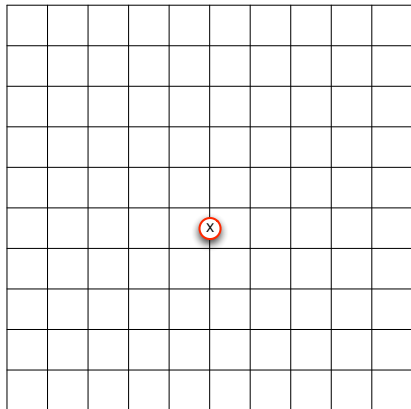
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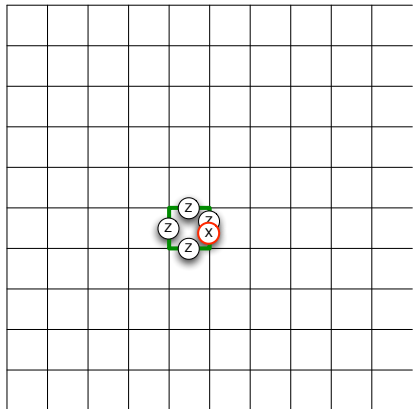
Particle creation

- Consider error $E = \sigma_X^i$.
- σ_X^i anti-commutes with adjacent plaquettes.
- $\sigma_X^i |\psi\rangle$ is a -1 eigenstate of B_p and $B_{p'}$
- Since $H = -(\sum_s A_s + \sum_p B_p)$, σ_X^i costs 2 energy units.
- This error has created a pair of magnetic particles.



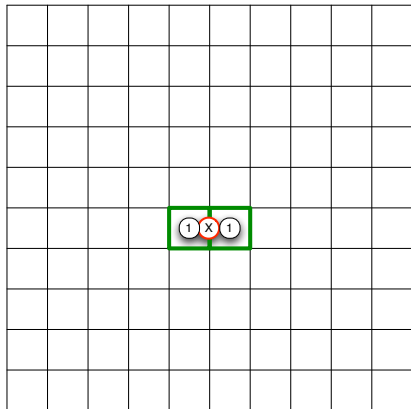
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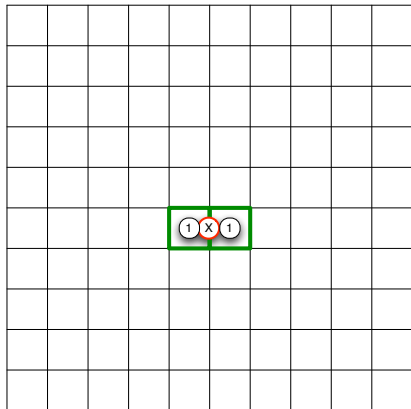
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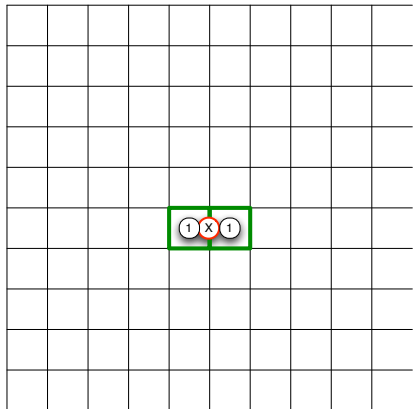
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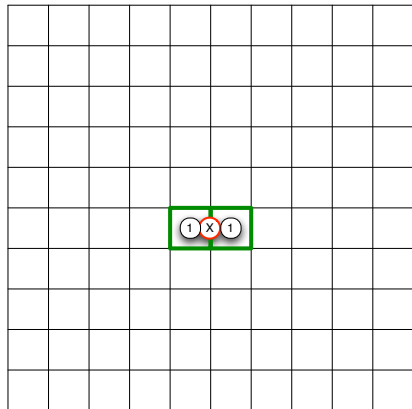


Particle diffusion

New error occurs on neighboring qubit:

- Restores the sign of the middle plaquette
- Flips the sign of the right plaquette

No net energy cost: particle has moved

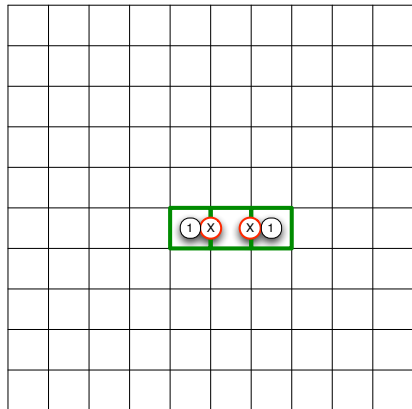


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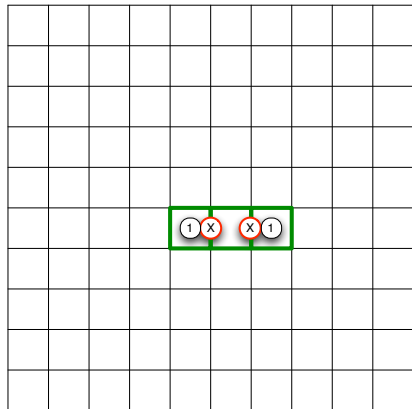


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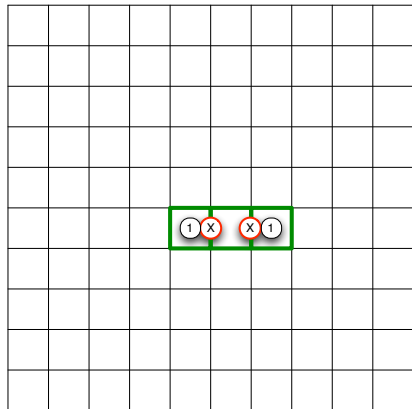


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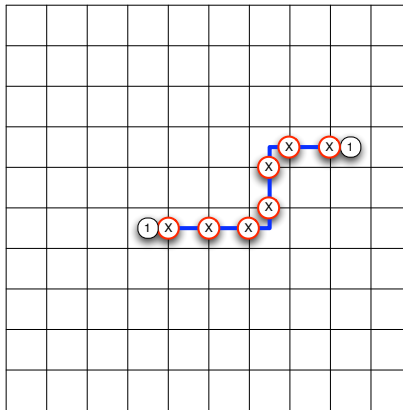
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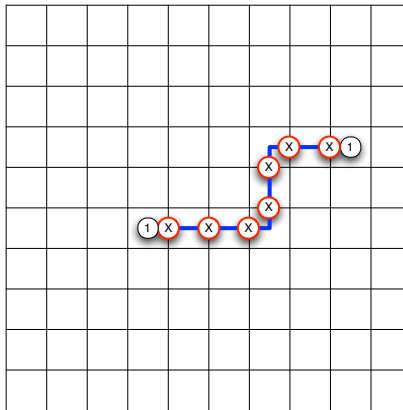
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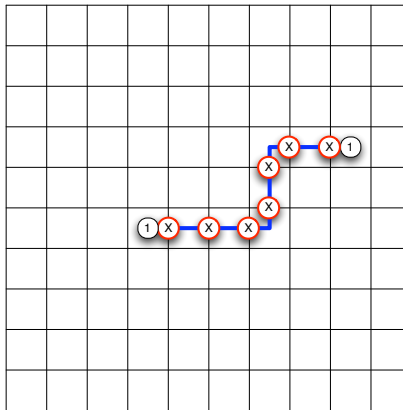
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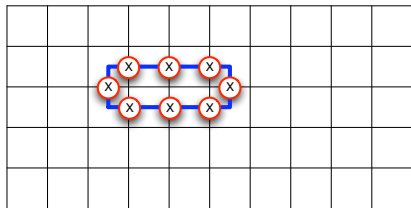
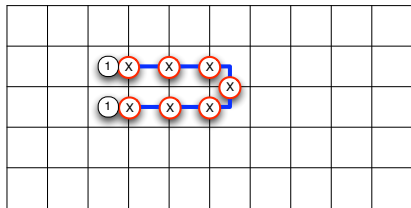
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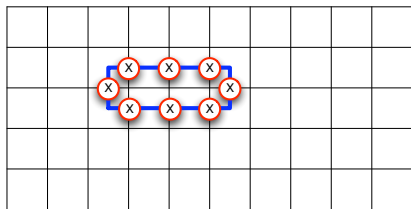
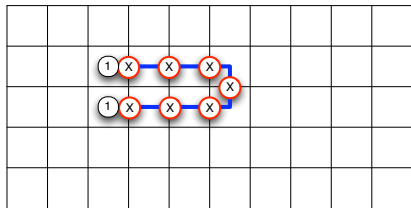
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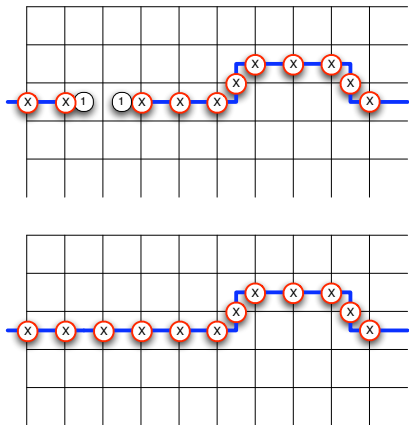
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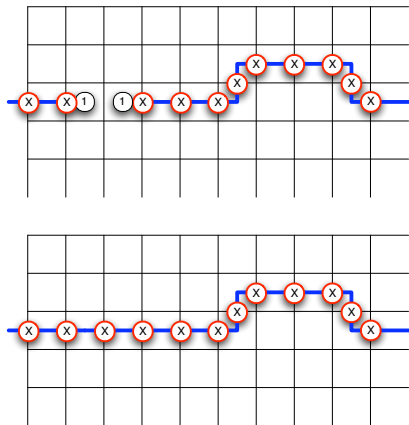
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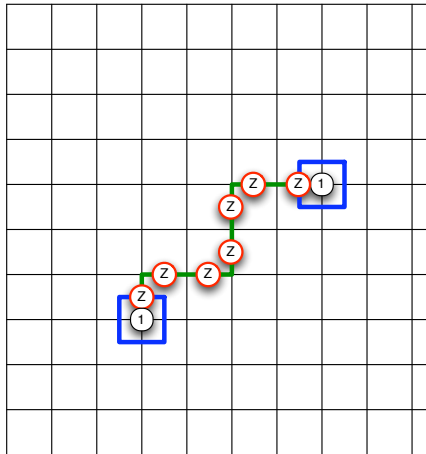
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Electrical particles

- The same story holds for σ_z errors
- These will create electrical particles located at the lattice's vertices (plaquette of dual lattice).



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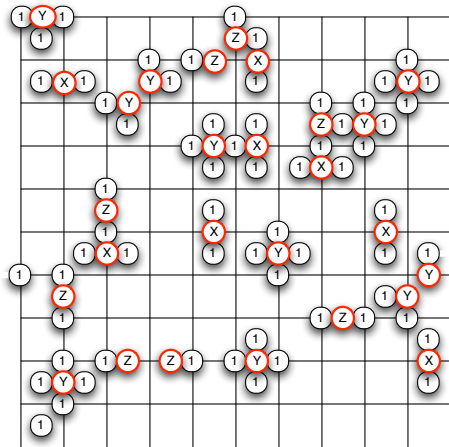
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Infer worldline homology from particle location.

15 % Noise rate



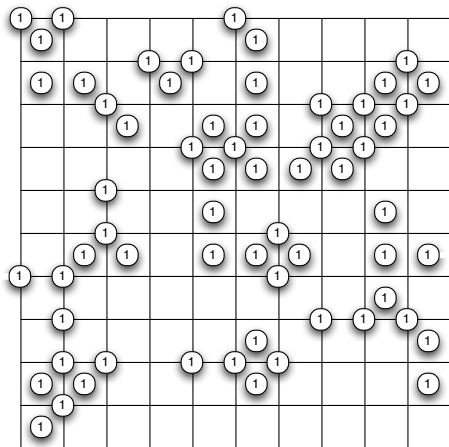
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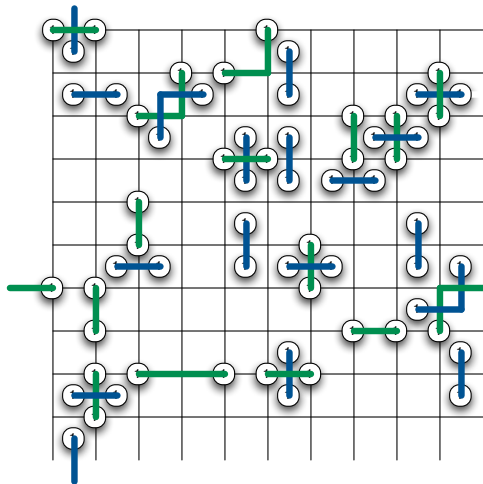
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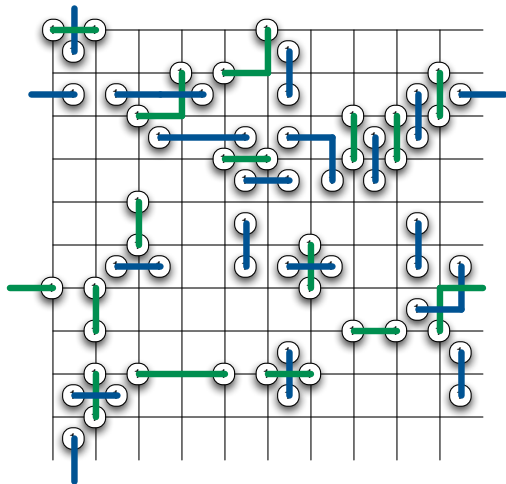
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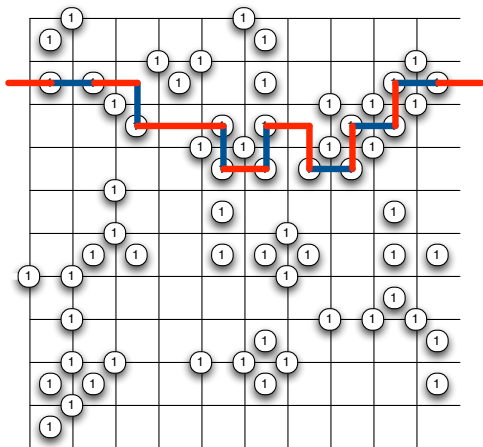


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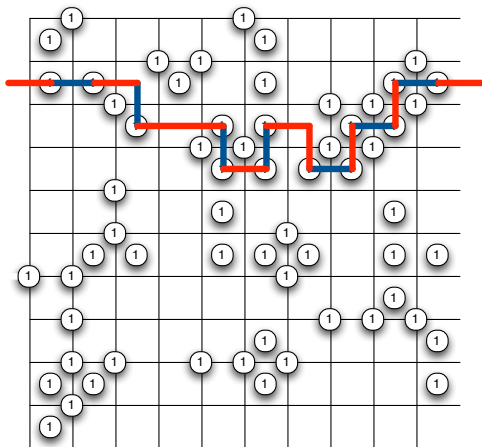
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- Find shortest path connecting all defects.
- Equivalent to minimizing energy of random bond Ising model.
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- It is very slow, $\mathcal{O}(\ell^6)$, limited to lattices $\ell \approx 100$.
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- No error with probability $1 - p$.
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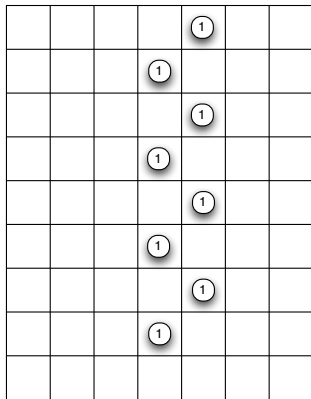
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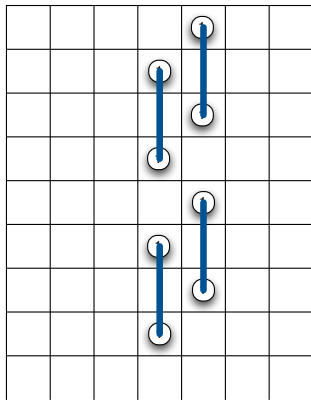
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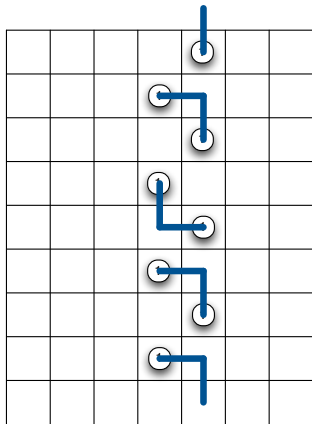
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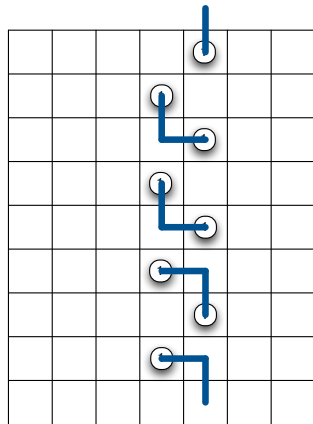
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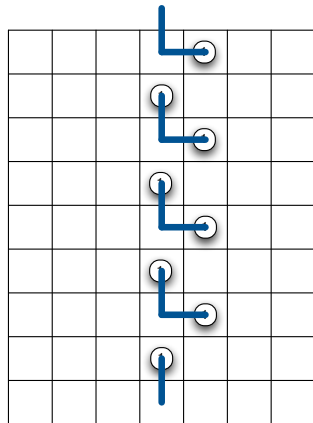
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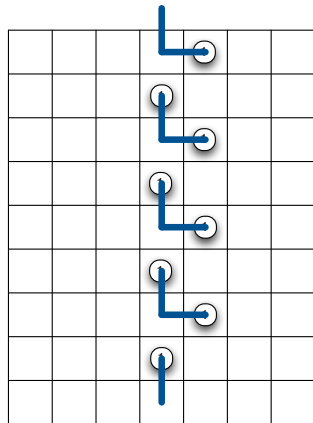
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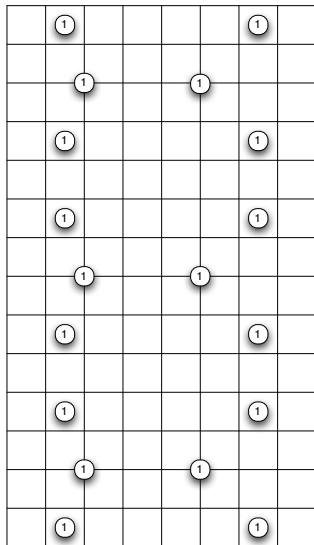
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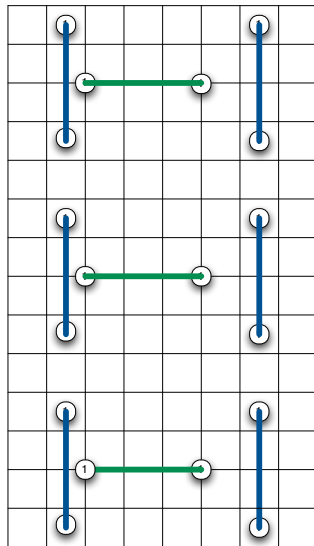
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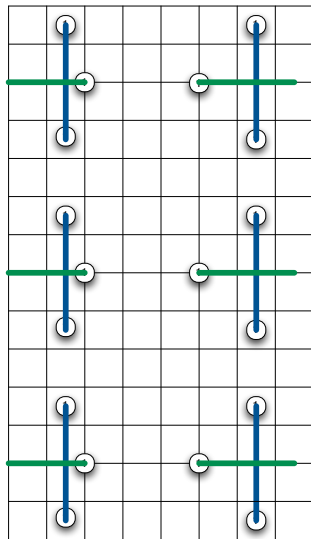
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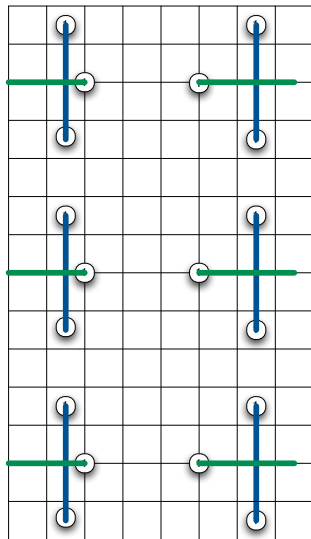
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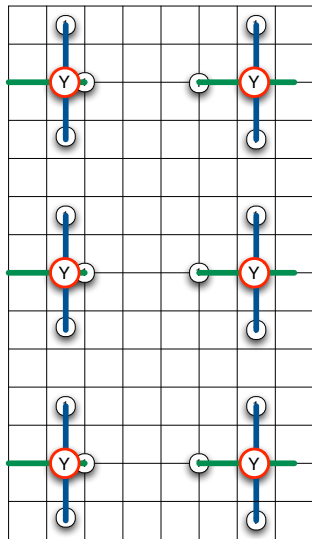
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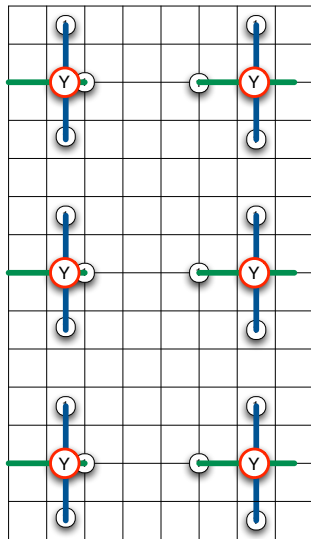
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- It is very slow, $\mathcal{O}(\ell^6)$, limited to lattices $\ell \approx 100$.
- It is not optimal:
 - Does not take into account the topological equivalence of errors.
 - Does not take into account correlations between magnetic and electric particles.
- Does not work with other topological codes.
 - E.g. color codes have 3-particle fusion channels.

Our approach

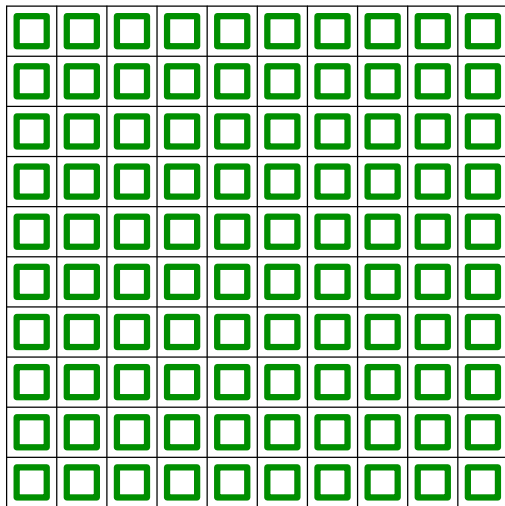
- Fast: complexity $\mathcal{O}(\ell^2 \ln \ell)$, parallelizable to time $\mathcal{O}(\ln \ell)$.
- Flexible and versatile:
 - Applies to different topological codes.
 - Allows tradeoffs between complexity and failure probability.
- Can achieve higher noise tolerance.

Outline

- 1 Kitaev's code
 - Definition
 - Hamiltonian's symmetries
- 2 Errors: defect creation, diffusion, and annihilation
- 3 Decoding problem
 - Task description
 - Perfect matching algorithm
- 4 Renormalization Group Decoder**
 - Coarse graining
 - Mean-Field Equations
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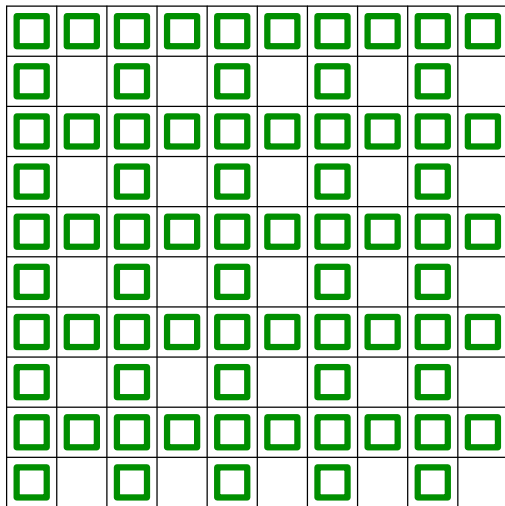
Scale invariance

- Original B_p checks
 - Basis change
(row operations on C)
 - Obtain scale invariant generators
 - Structure similar to a concatenated code.
- Soft-decode each small block.
 - Pass information to next encoding level.



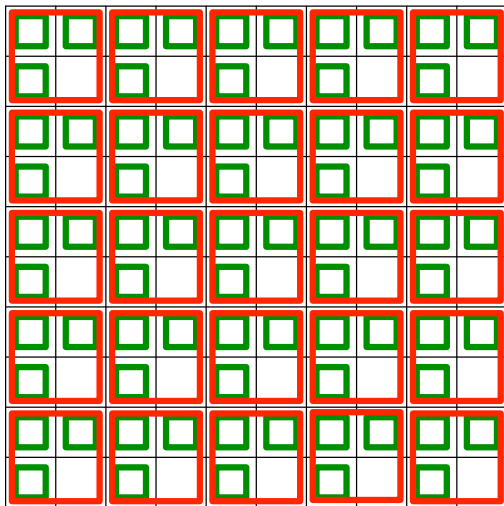
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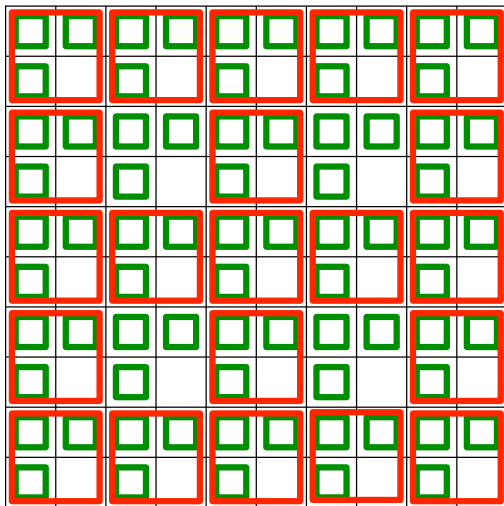
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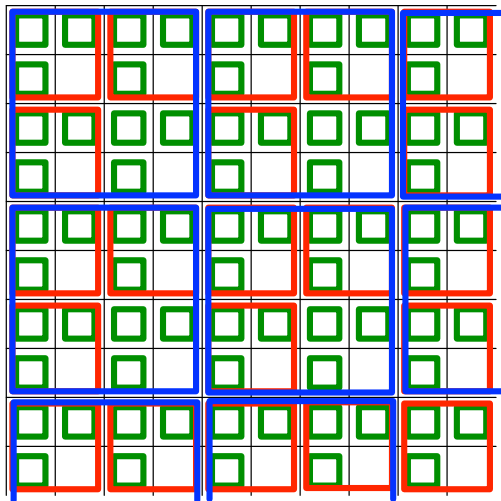
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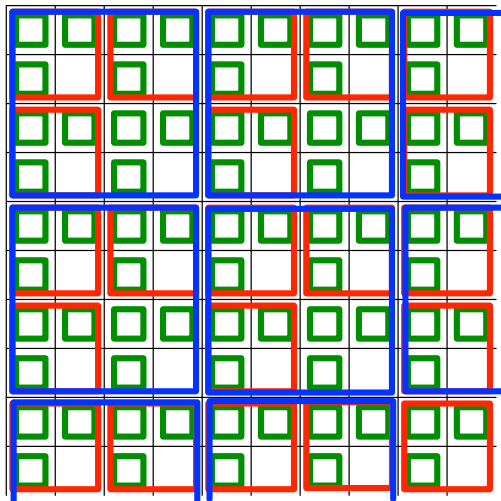
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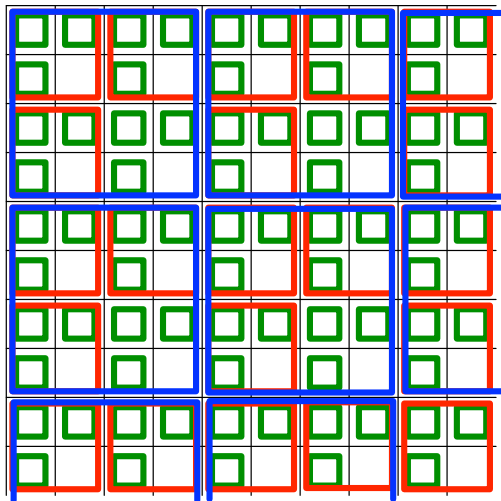
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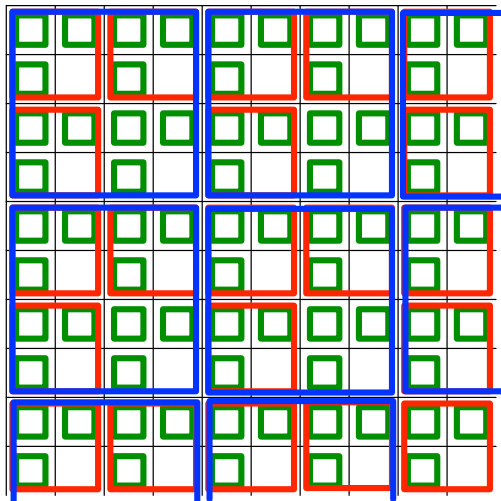
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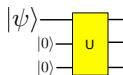
Concatenated code

- Encoding one qubit in 3 qubits.

Decoding

$$P(L) = \sum'_E P(E) \text{ where}$$

- Sum over E equivalent to L and with right syndrome.
- $P(E)$ given by error model.
- Encode each of these qubits...



Decoding

- Compute error probability for each encoded qubit.
- Pass that probability to the next level up.

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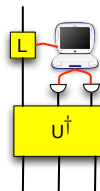
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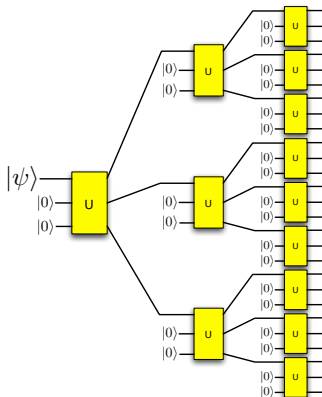
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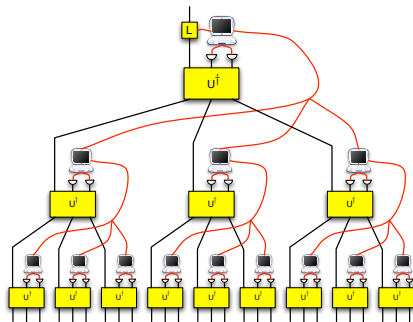
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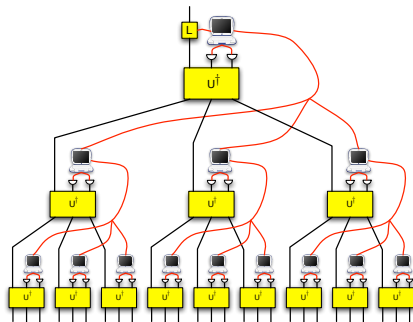
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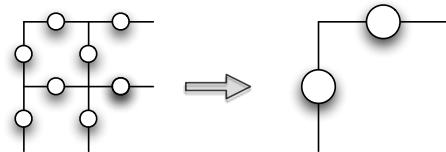
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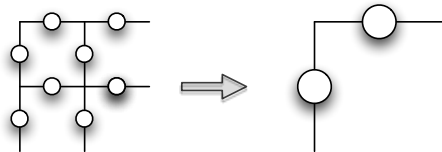


Renormalisation: information coarse graining



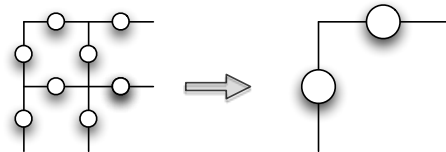
- Think of the Kitaev's code as a concatenated code:
 - It is made up of a bunch of small (open boundary) topological codes, joined into larger topological codes, etc.
- Given the particle configuration in a unit cell, compute the prob.
 - That it is transversed from top to bottom by a magnetic particle.
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 - etc.
- There are 16 possibilities corresponding to $\{I, X, Y, Z\}^2$, 2 qubits.
- This is done by brute force: sum over all worldline configurations.

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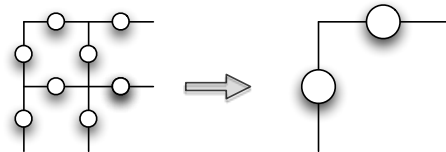
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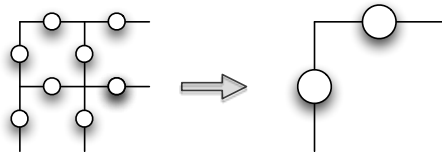
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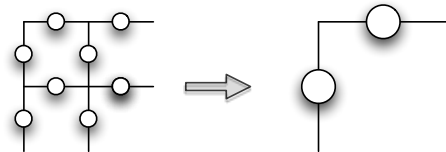
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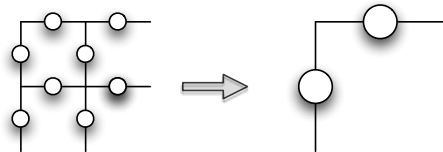
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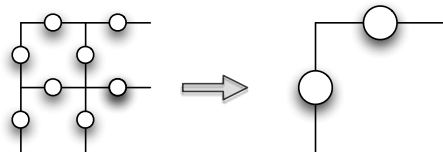
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Overlapping cells

Topological codes are NOT concatenated codes.

- Unit cell does not enclose enough stabilizers.
 - Area law increase of parameter space.
- Use overlapping cells instead.



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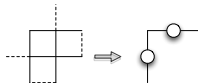
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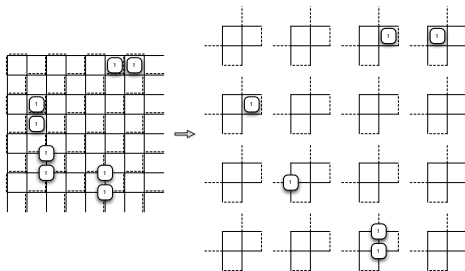
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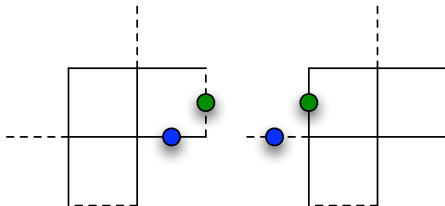
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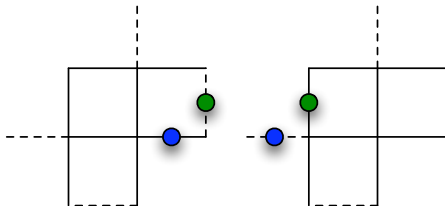


Self-consistence



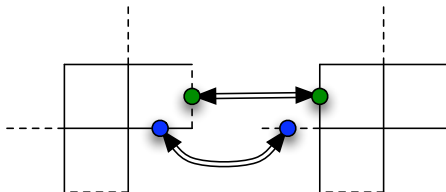
- Boundary qubits treated as independent variables on neighboring unit cells.
- Probabilities assigned by different cells to a given qubit differ.
- Impose mean-field consistency conditions on marginal probabilities.
- Solve by belief propagation.
- Complexity $\mathcal{O}(\ell^2)$ parallelizable to constant time.

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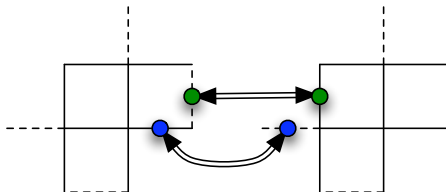
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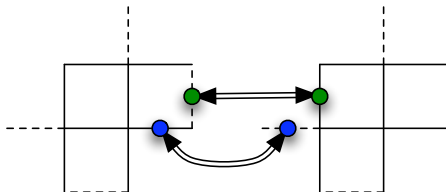
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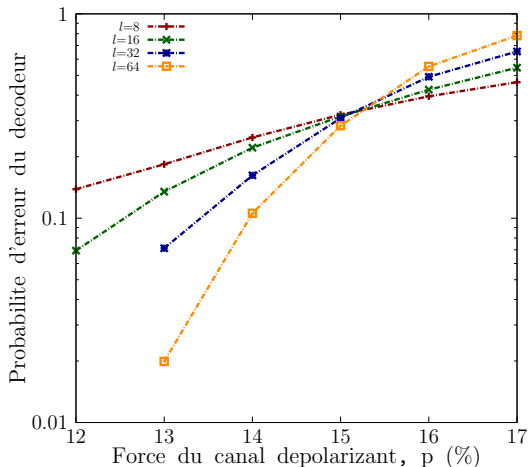


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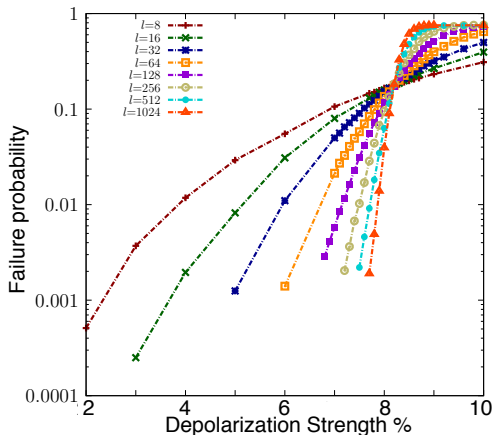
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RG decoder with 3 BP rounds



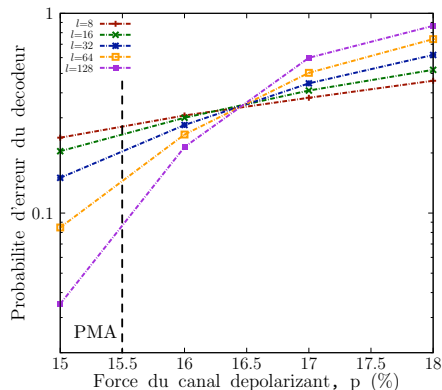
- Threshold $\approx 15\%$, compared to 15.5% for PMA.
- Exponentially faster with marginal performance loss.

Smaller unit cell, 2×1



- Bit-flip threshold $\approx 8.2\%$, compared to 10.3% for PMA.
- Much faster even without parallelization (10^6 sites).
- Illustrates flexibility.

Higher threshold



- Use of additional belief propagation.
- Threshold $\approx 16.5\%$, compared to 15.5% for PMA.
- Illustrates flexibility.

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Topological phase

- Conventional phases of matter are classified by order parameters and symmetry breaking (Landau-Ginsberg)
- How to characterize and classify topological phases?
- Chen, Gu, and Wen: local equivalence of ground-state manifold
 - $UP_0U^\dagger = P'_0$
 - If $|\sigma| \leq r$, then $|U\sigma U^\dagger| \leq w + c$ (finite light cone).
 - P_0 and P'_0 are adiabatically connected.
 - Hard to determine (not constructive).

If two topological codes are in the same phase

- Switch between them fault tolerantly during computation
- Virtually switch between them for the sake of decoding
 - Error model remains local
 - Stabilizers remain local, reliable error syndrome

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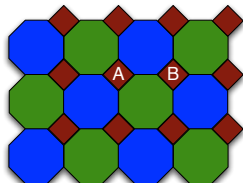
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 - Hard to determine (not constructive).

If two topological codes are in the same phase

- Switch between them fault tolerantly during computation
- Virtually switch between them for the sake of decoding
 - Error model remains local
 - Stabilizers remain local, reliable error syndrome

Topological color code

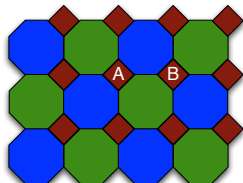
- Qubits located on vertices
- Plaquette stabilizers $S_p = \bigotimes_{j \in \partial p} \sigma_j$, for $\sigma = \sigma^x$ and σ^z .
- 16 topological charges
 - 10 bosons
 - 6 fermions
- Same particle type and statistics as 2 copies of Kitaev's code



- Obtained by identifying topological charges
- Local stabilizers $\Rightarrow$ Local stabilizers
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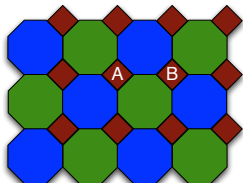
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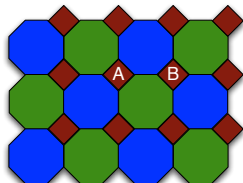
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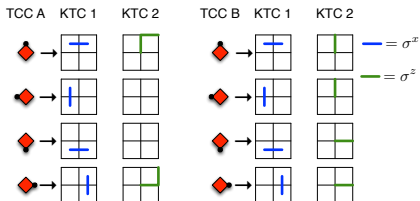
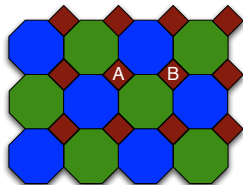
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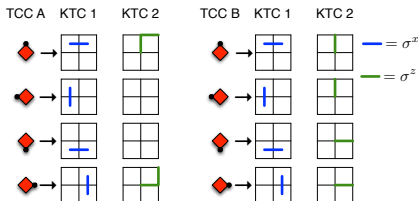
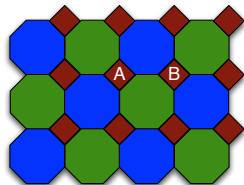
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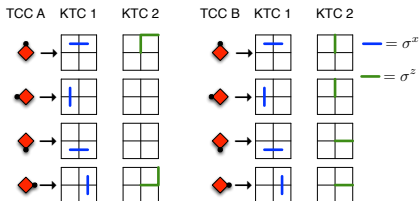
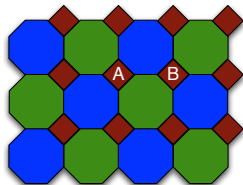
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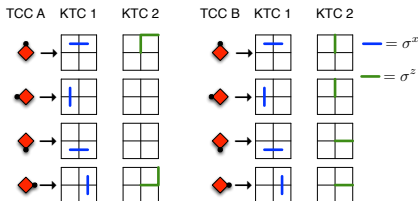
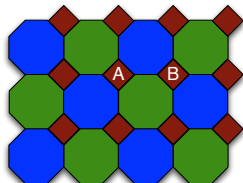
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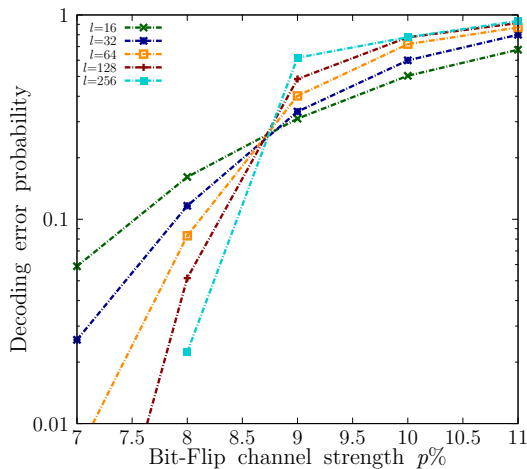
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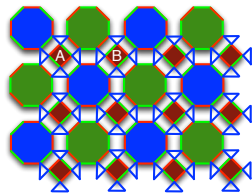
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Results



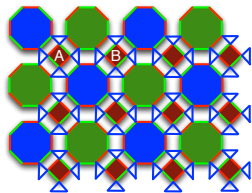
Topological subsystem code

- Qubits located on vertices
- Gauge generators on edges $\sigma \otimes \sigma$
 - $\sigma = \sigma^x, \sigma^y, \sigma^z$ red, green blue
- Stabilizers = center of gauge group
 - Has local generators
- Transversal Clifford group
- Only 3 semionic fermions
- Matches a subset of $2 \times$ Kitaev's code
 - $f_1 \leftrightarrow (f, 0)$
 - $f_2 \leftrightarrow (e, f)$
 - $f_3 \leftrightarrow (m, f)$



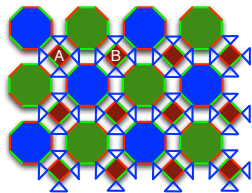
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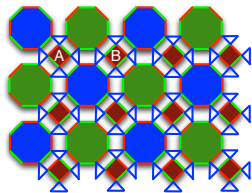
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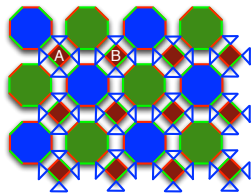
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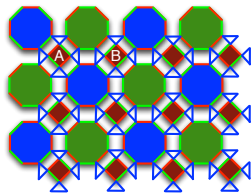
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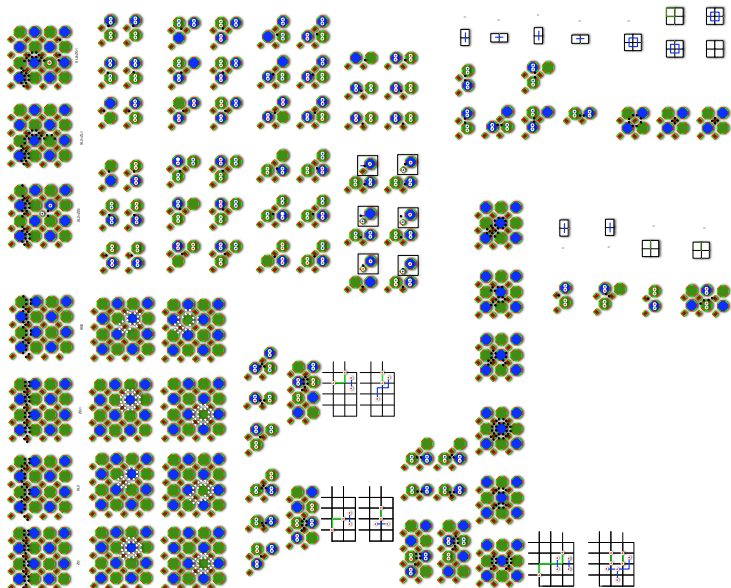


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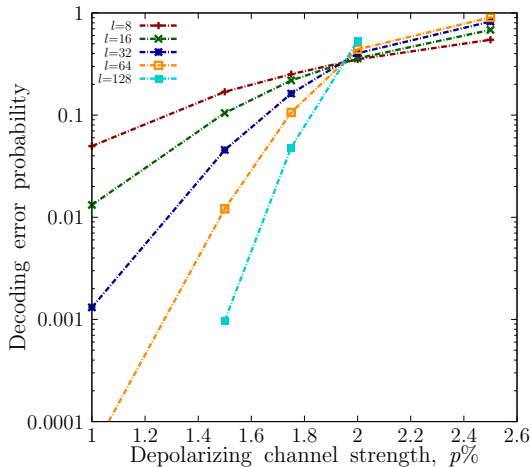
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Mapping



Results



- Decoding problem: infer defect worldline homology from "snapshots" of their configuration.
- Equivalent to minimizing free energy of some spin model.
 - Minimizing energy can be done efficiently for Kitaev's code (RBIM).
 - Suboptimal, slow, and only suited to this model.
- RG decoding algorithm
 - Exponentially faster.
 - Versatile (other codes, time/performance tradeoff).
 - Higher threshold.
 - Heuristic (Bravyi has proved a threshold... 10^{-22})
- Local equivalence between codes
 - Defines topological phases
 - Universality of decoding algorithms
 - Enhanced fault tolerance ?
 - All stabilizer codes topologically equivalent to Kitaev?

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